

# Physical approach to price momentum and its application to momentum strategy

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## Abstract

We introduce various definitions for price momentum of financial instruments in quantitative and mathematical ways. Measurement of the price momentum derived from the concept of momentum in physics can be conducted with velocity and mass defined in diverse manners. By using the physical momentum of price as a selection criterion, the momentum/contrarian strategies are implemented with equities in the South Korean stock markets including the KOSPI 200 and its subuniverses. The physical momentum strategies provide better expected returns and risk-reward ratios than those of the traditional momentum strategy at many lookback-holding pairs in weekly scales and short terms in monthly scales. In addition to that, the spontaneously symmetry breaking of arbitrage is also tested for the physical momentum strategies and the strategies with a scheme from the symmetry breaking of arbitrage generate the stronger performance and increase stability of the portfolios than the strategies without the scheme.

*Keywords:* price momentum, momentum/contrarian strategies, spontaneous symmetry breaking of arbitrage

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## 1. Introduction

Searching for existence of arbitrage is an important task in finance. In the case of systematic arbitrages, regardless of their origins such as market microstructure, firm-specific news/events, and macroeconomic factors, it is possible to exploit arbitrage opportunities via trading strategies in order to take persistent profits. Among such kinds of systematic arbitrage chances, they are also called as market anomalies, if origins of them are not well-explained nor understood quantitatively and qualitatively [1, 2]. To academic researchers in finance, it is very useful to testing robustness of the efficient market hypothesis

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[3, 4] and no-arbitrage theorem. Although they had played the keystone roles in asset pricing theory and general finance, their statuses have been changed as alternative theories that intrinsically allow the pricing anomalies have appeared in financial markets, as instances, the adaptive market hypothesis [5, 6, 7] and behavioral finance [8, 9, 10, 11]. Hunting for the systematic arbitrage opportunities is also crucial to market practitioners such as traders and portfolio managers on the Wall street because it is the core of money-making process that is the most important role of them.

Among these market anomalies, the price momentum effect has been the most well-known example to both groups. Since Jegadeesh and Titman’s seminal paper [12], it is reported that prices of financial instruments have the momentum effect that the future price movement tends to keep the same direction along which it has moved during a given past period. It is also realized that the momentum strategy, a long-short portfolio based on the momentum effect, has been a profitable trading strategy in stock markets of numerous developed and emerging countries during a few decades even after its discovery [13, 14]. In addition to the existence in equity markets, other asset classes such as foreign currency exchange [15], bond [16], futures [16, 17], and commodities markets [18] also have the momentum effect large enough to be implemented as the trading strategy.

In spite of its success in profitability over diverse asset classes and markets, its origin has not been fully understood in the frame of traditional mainstream finance. This is why the momentum effect is one of the most famous market anomalies. Attempts to explain the momentum effect in methods of factor analysis have failed [19] and the reason why the momentum effect has persisted over decades still remains mysterious. The Fama-French three factor model is able to explain only parts of the momentum return [19]. The lead-lag effect or auto-/cross-sectional correlation between equities are one of the possible answers to the momentum effect [20, 21]. The sector momentum is another partial interpretation on the anomaly [22]. Additionally, behavioral aspects of investors such as individual and collective responses to financial news and events have broadened the landscape of understanding on the momentum effect [23, 24, 25, 26]. A transaction cost is also considered as a factor caused the momentum effect [27]. Unfortunately, none of these explanations are capable of providing the entire framework for explaining why the momentum of price dynamics exists in many financial markets.

Not only demystification on the origins of the price momentum, pursuit on the profitability and implementability of the momentum effect in the markets also has been interesting to academics and practitioners. For example, although several studies [28, 29, 30, 31, 32] found that the momentum strategies in some Asian markets such as the Japanese and Korean stock markets are not profitable, Asness et al. [16] discovered that the momentum strategy in Japan becomes lucrative, when it is combined with other negatively correlated strategies such as value investment. Not only in several stock markets, the hybrid portfolio of value and momentum also outperforms each of the value and momentum portfolios across the assets. Their study paid attention to implementation of

the momentum strategy combining fundamental value investment indexes such as book-market (BM) ratio<sup>1</sup> which also has been used to unveil the origins of the momentum effect in Fama-French three factor analysis. In other words, their work can be understood as construction of the hybrid portfolio to increase the profitability and stability of the portfolios based on the momentum strategy. Moreover, the selection criteria for the hybrid portfolio are considered as the multiple factors related to the momentum returns whether they are positively correlated or negatively correlated. Academically, this observation has the important meaning in the sense that these multiple filters can explain their contributions to the momentum returns. In practical viewpoint, it is definitely the procedure for generating trading profits in the markets.

Another method for improving profitability of the momentum strategy is introducing various selection criteria to construct the momentum portfolio. First of all, simple variation on the original momentum selection rule can be made. Moskowitz et al. [17] also suggested new trading strategies based on time series momentum which constructs the momentum portfolios by time series regression theory. It is not simply from a cumulative return during a lookback period as a sorting variable but from an autoregressive model of order one which can forecast the future returns under given conditions such as the past returns and volatilities. The forecasted return is used as the selection criterion for the time series momentum strategy. The time series momentum strategy performs very nicely even during market crisis. It also shares the common component which drives the momentum return with the cross-sectional momentum strategy across many asset classes. This fact imposes that the momentum strategy is improved by the modified cumulative return criterion and there is possibility to find the better momentum strategies in performance and stability.

Besides only considering the cumulative return, introduction of other kinds of proxies for the portfolio selection rules has been also worth getting attention. George and Hwang [33] used 52-week high price<sup>2</sup> as a selection criterion and the momentum portfolio based on the 52-week high price generated stronger returns. Additionally, tests with doubly-sorted portfolios, which are constructed by the cumulative return or sector momentum and the 52-week high price, exhibit the superiority of the 52-week high price criterion. The factor analysis also shows that the return from the 52-week high price factor is not only stronger than the traditional or sector momentum factors but also statistically more significant and important in the momentum return modeling. The dominance of 52-week high momentum criterion is also observed in the various international stock markets [32].

Risk metrics are also able to serve as the ranking criteria. Rachev et al. [34] used the risk measures as the sorting criteria for their momentum portfolios

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<sup>1</sup>It is also related to price-book (PB) ratio inversely. Many literatures on momentum mostly use BM ratio as a momentum-driven factor and PB ratio also known as PBR is frequently mentioned in fundamental analysis of stocks.

<sup>2</sup>The 52-week high price is the highest price during last 52 weeks, i.e. 1 year.

instead of the raw returns over estimation periods. In their work, Value-at-Risk, Sharpe ratio, R-ratio, and STARR ratio were used as the alternative ranking rules. In the S&P 500 universe during 1996–2003, their momentum portfolios from the risk metrics provided the better risk-adjusted returns than the traditional momentum strategy which uses the cumulative return as a sorting rule. In addition to that, the new momentum portfolios had lower tail indexes for winner and loser portfolios. In other words, these momentum strategies based on the risk metrics obtained the better risk-adjusted returns with acceptance of the lower tail risk.

Back to physics, the momentum in price dynamics of a financial instrument is also an intriguing phenomenon because the persistent price dynamics and its reversion can be understood in terms of inertia and force. The selection rules of the momentum strategy is directly related to the ways of how to define and measure “physical” momentum in price dynamics of the instrument. When the instrument is considered as a particle in an one-dimensional space, the price momentum is also calculated if mass and velocity are defined. Since the momentum effect exists, it can be concluded that price of an equity has an inertia that makes the price keep their direction of movements until external forces are exerted. This idea is also able to explain why the cumulative return based momentum strategy generates the positive expected returns. However, it has been not much attractive to physicists yet. Most of the econophysics community doesn’t have been interested in trading strategy and portfolio management so far.

Recently, Choi [35] suggested that the trading strategy can be considered as being in the spontaneous symmetry breaking phase of arbitrage dynamics. In his work, the return dynamics had an inversion symmetry in the return which can be broken by selection of the ground state. When a control parameter is smaller than a critical value, the strategy is in the arbitrage phase and we expect the non-zero expected return which is not permitted in the efficient market hypothesis. Random fluctuation around the non-zero value makes variance of the strategy return and the risk of loss is still existing. The important caveats were not only that the arbitrage strategy can theoretically generate the non-zero expected returns which are emergent from the symmetry breaking concept but also that the idea is empirically meaningful when it is applied to real trading strategies. In the simple simulation, the control parameter which triggers phase transition was estimated from an autocorrelation coefficient of the strategy return time series. The weekly momentum strategy was executed based on the scheme using the symmetry breaking arbitrage. If the strategy is expected to be in the arbitrage phase, the strategy is exploited and if not, the execution is stopped. The momentum strategy with the scheme had the better expected return and Sharpe ratio than the traditional momentum strategy does.

In this paper, we introduce various definitions for the physical momentum of equity price. Based on those definitions, the equity price momentum can be obtained from real historical data in the South Korean stock markets, especially from the KOSPI 200 components, the major 200 equities in the market. After computing the physical momentum, implementation of the momentum

strategies based on the candidates for the price momentum increases the validity of our approach to measuring the physical momentum in equity price. Empirically, these new candidates for the selection criterion originated from the physical momentum idea provide the better returns and Sharpe ratios than the original criterion, i.e. the raw return. The structure of this paper is the following. In the next chapter, the definition of velocity in equity price space and possible candidates for mass are introduced and then the price momentum is defined with the financial velocity and mass. In section 3, we briefly inform the fundamentals of the momentum strategy and specify the datasets used for our analysis. In section 4, results for the physical momentum strategies are given. In section 5, the physical momentum strategies are tested for the symmetry breaking arbitrage proposed by Choi [35]. In section 6, we conclude the paper.

## 2. Theoretical background

If an one-dimensional space for price of a financial instrument is introduced, it is possible to consider that the price is in motion on the positive half-line. Although the negative price is conceptually proposed by Sornette [36], the negative price of the instrument is not allowed in practice.<sup>3</sup> The price dynamics of the financial instrument is now changed to an one-dimensional particle problem in physics. To extend the space to the entire line, the log price is mapped to the position  $x(t)$  in the space by

$$x(t) = \log S(t)$$

where  $S(t)$  is the price of the instrument. This transformation is not new to physicists because Baaquie [37, 38] already introduced the same transformation to derive path integral approach to option pricing theory. Baaquie used the transformation in order to find the relation between the Black-Schole equation and Schroedinger equation. With this re-parametrization, an option pricing problem was transformed to an one-dimensional potential wall problem in quantum mechanics. However, it was not for introducing the physical momentum concept mentioned above. With the log return,  $x(t)$  covers the whole line from the negative to positive infinity. In addition to the physical intuition, the log price has some advantages in finance. First of all, it is much simpler to calculate the log return from the log price because the difference of two log prices is the log return. Contrasting to the log return, the raw return is more complicated to compute than the log return. Secondly, one of the basic assumptions in mathematical finance is that the returns of financial instruments are log-normally distributed and we can handle normally-distributed log returns.

Having the advantages of the log price described above, it is natural to introduce a concept of velocity into the one-dimensional price space. In the

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<sup>3</sup>Sornette not only pointed that the negative equity price is introduced only for symmetry breaking but also explained why the negative price is not observed under real situations using dividend payment as an external field in symmetry breaking.

case of the log price, the log return  $R(t)$  is expressed in  $x(t)$  by

$$\begin{aligned} R(t) &= \log S(t) - \log S(t - \Delta t) \\ &= \frac{x(t) - x(t - \Delta t)}{t - (t - \Delta t)} \\ &= \frac{\Delta x(t)}{\Delta t}. \end{aligned}$$

In the limit of infinitesimal time interval ( $\Delta t \rightarrow 0$ ), the log return becomes

$$R(t) = \frac{dx(t)}{dt} = v(t)$$

where  $v(t)$  is the velocity of the instrument in the log price space,  $x(t)$ . When the mapping between the log price and position in the space is introduced, it imposes the relation between the log return and velocity. Although this relation works only in the limit of  $\Delta t \rightarrow 0$ , it can be applied to the discrete time limit if the length of the whole time series is long enough to make the time interval relatively shorter.

The cumulative return  $r(t)$  is expressed in  $v(t)$  by

$$\begin{aligned} r(t) &= \frac{S(t) - S(t - \Delta t)}{S(t - \Delta t)} = \exp(R(t)) - 1 \\ &= v(t) \left( 1 + \frac{1}{2}v(t) + \cdots + \frac{1}{n!}(v(t))^{n-1} + \cdots \right). \end{aligned}$$

Since the log return is usually small such as  $|v(t)| \ll 1$  in real data, higher-order terms in  $v(t)$  can be treated as higher-order corrections on  $r(t)$  and it is possible to ignore the higher-order corrections if  $|v(t)| \ll 1$ . In this sense, the cumulative return can be approximated to  $v(t)$ . However this relation is broken in the cases of heavy tail risks caused by financial crisis or firm-specific events such as bankruptcy, merger and acquisition (M&A), and good/bad earning reports of the company because  $|v(t)|$  can be comparable to one or greater than one and the higher-order perturbations should be considered.

Based on this correspondence, the concept of price momentum can be quantified similar to the classical momentum in physics by

$$p = mv$$

where  $m$  has the same role to physical mass. In particular, when velocity is given in the log return, contribution of mass to the price momentum can be expressed in the following way,

$$\begin{aligned} p &= m \log(1 + r) \\ &= \log(1 + r)^m. \end{aligned}$$

The financial mass  $m$  plays a role of amplifying the price change as the mass becomes larger. This amplification is understood as filtering of market information on price. Some instruments are heavily influenced by the investors'

psychology and other market factors but other don't. In this sense, the mass can act a role of the filter which is unique to each instrument and encodes the instrument-specific characters. This interpretation is also well-matched to the physical analogy that mass is a physical constant which is unique to each particle. The original ranking criterion in the traditional momentum strategy is a special case of this momentum definition. In the cumulative return momentum strategy, it is assumed that each of equities has the identical mass,  $m = 1$ . However, the identical mass assumption seems not to be reasonable because each equity has distinct properties and shows inherent price evolutions. In order to capture these heterogeneities between characteristics of each equity, escaping from the identical financial mass for all equity is more natural and introduction of the financial mass concept to the momentum strategy look plausible. Although the physical momentum concept can be applied to other asset classes, we focus on only the equities in this paper.

As described in the previous paragraph, the financial mass can convey the instrument-specific information. However, it is obvious that all kinds of information cannot work as candidates of the mass because it should be well-matched to intrinsic properties of physical mass. In this sense, liquidity is a good candidate for the financial mass. Its importance in finance is already revealed in many financial literatures in terms of volume or turnover rate. [39, 40, 41, 42, 43]. In particular, Datar [39] reported that the past turnover rate is negatively correlated to the future return. Under the same size of the momentum, the larger turnover rate brings the poorer future return i.e. the illiquid stocks have higher expected returns. Even after controlling other factors like firm-size, beta<sup>4</sup>, and BM ratio, the past turnover rate has the significant negative correlation with the future return. It is possible to understand that the trading volume incorporates integrated opinions of investors and makes the price approach to the equilibrium asymptotically. In the viewpoint of information, trading can be understood as the exchange of information between investors with inhomogeneous information. More transactions occur, more information is widely disseminated over the whole market and the price change becomes more meaningful. Lee and Swaminathan [42] also provided the similar result that stocks with low past trading volumes tend to have high future returns. Additionally, the study found that the momentum strategy among high volume stocks is more profitable. The similar result is obtained in the South Korean market [43].

The possible mass candidates which are also well-matched to the analogy of physical mass are the volume, total transaction value, and inverse of volatility. If the trading volume is larger, the price movement can be considered the more meaningful signal because the higher volume increases the market efficiency. The amount of the volume is proportional to mass  $m$ . As mentioned in the previous paragraph, the relation between the trading volume and asset return is already studied in finance [39, 41, 42]. Instead of the raw volume, we need

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<sup>4</sup>The beta in finance is the correlation between the price and benchmark scaled by the market variance.

to normalize the daily volumes with the total number of outstanding shares and this normalized value is also known as a turnover rate. The reason of this normalization is that some equities intrinsically have the larger trading volumes than others because the total number of shares enlisted in the markets are much larger than other equities or because they get more investors' attentions which cause more frequent trades between investors. The share turnover rate, trading volume over outstanding shares is expressed in  $v$  in the paper.

Similar to the volume, the daily transaction value in cash can be used as the financial mass. If an equity on a certain day has the larger transaction value, investors trade the equity frequently and the price change has more significant meanings. Additionally, the transaction value contains more information than the volume. For examples, even though two equities have the same daily volumes and daily returns on a given day, the higher priced equity has the larger trading value if two prices are different. The more important meaning is that even though market information such as close price, volume, return, and price band are identical, the trading value in cash can be different. As an instance, when one equity is traded more near the lowest price of the daily band but the other is traded mainly around the daily highest price region, the total transaction values of two equities are definitely different. It also needs to be normalized because each equity price is different. The normalization of dividing total transaction value by market capitalization is expressed in  $\tau$  in the paper.

The return volatility  $\sigma$  is inversely proportional to the financial mass  $m$ . If the volatility of a certain equity in a given period is larger, the equity price is easy to fluctuate severely than other equities with the smaller volatilities. This correspond to the situation in physics that a lighter object can move more easily than a heavy object under the same force. This definition of the financial mass is also matched with the analogy used in Baaquie's works [37, 38]. In his works, the Black-Scholes equation was transformed into Hamiltonian of a particle under the potential which specifies the option. The mass of a particle in the Hamiltonian was exactly same to the inverse of the return volatility. Since the volatility is also interesting to economists and financiers, there are large number of literatures which cover the link between volatility and return [44, 45].

With the fractional volume and fractional transaction value as the proxies for the mass, it is possible to define two categories of the physical momentum,

$$p_{t,k}^{(1)}(m, v) = \sum_{i=0}^{k-1} m_{t-i} v_{t-i}$$

and

$$p_{t,k}^{(2)}(m, v) = \frac{\sum_{i=0}^{k-1} m_{t-i} v_{t-i}}{\sum_{i=0}^{k-1} m_{t-i}}$$

over the period of the size  $k$ . The latter one is reminiscent of the center-of-mass momentum in physics and the similar concept is used as the embedded



capital gain in Grinblatt and Han [46]. Since two different categories for the momentum calculation, two for return, and two for mass are available, there are eight different momentum definitions for an equity.

It is easily found that the cumulative return can be expressed in  $p^{(1)}$  by

$$\begin{aligned} r_{t,k} &= \exp\left(\sum_{i=0}^{k-1} R_{t-i}\right) - 1 = \exp(p_{t,k}^{(1)}(1, R)) - 1 \\ &\approx p_{t,k}^{(1)}(1, R) + \mathcal{O}\left((p_{t,k}^{(1)}(1, R))^2\right) \end{aligned}$$

an this shows that the traditional momentum in finance is a special case of the physical momentum. In this sense, let's call  $r_{t,k} = p_{t,k}^{(0)}$ . In addition to that, since exponential function and log function are strictly increasing functions, the mapping between  $p_{t,k}^{(0)}$  and  $p_{t,k}^{(1)}(1, R)$  is one-to-one.

Since the return volatility over the period has more practical meanings than the sum of daily volatilities during the period, the third class of the physical momentum is defined by

$$p_{t,k}^{(3)}(m, v) = \bar{v}_{t,k} / \sigma_{t,k}$$

where  $\bar{v}_{t,k}$  is the average velocity at time  $t$  during the past  $k$  periods. There are also two different definitions for  $p_{t,k}^{(3)}$  computed from the normal return and log return. This is closely related to the Sharpe ratio,  $SR$ ,

$$SR = \frac{\mu(r - r_f)}{\sigma(r - r_f)}$$

where  $r_f$  is the risk-free rate. If the risk-free rate is small and ignorable,  $p_{t,k}^{(3)}$  approaches to the Sharpe ratio. The momentum strategy with this ranking criterion is reminiscent of the Sharpe ratio based momentum strategy by Rachev et al. [34]. Similar to the Sharpe ratio,  $p_{t,k}^{(3)}$  can be related to the information ratio that uses excessive returns over the benchmark instead of the risk-free rate in the definition. However, we don't consider the risk-free rate nor the benchmark return as a reference point of the portfolio returns in this paper.

With  $p_{t,k}^{(1)}$ ,  $p_{t,k}^{(2)}$ , and  $p_{t,k}^{(3)}$ , total eleven different definitions of physical momentum including the traditional cumulative return are possible candidates for the physical equity momentum. Each of them is originated from the physical and financial foundations. Additionally, they are relatively easier to quantify than other risk measures used in Rachev's work [34]. Although it is possible to consider more complicated functions of other market data for the price momentum, it is beyond the scope of this paper.

### 3. Application to real data

#### 3.1. Dataset

The major market universe for the study is the KOSPI 200 index, one of the main stock market indexes in the South Korean market. It is the value-

weighted index of 200 equities which represent industry sectors and which covers from small to large market capitalization companies. The importance of the KOSPI 200 is that the KOSPI 200 is the only stock market index which has derivatives in the South Korean market. Since the South Korean derivative markets for options and futures are ones of the most liquid derivative markets in global capital markets, the index and its components are heavily correlated with the movement of derivative markets, and vice versa. Besides that, the KOSPI 200 is considered as the benchmark index by many mutual funds because it is the best index that can reflect sentiment of the entire market. Additionally, it has numerous exchange-traded funds (ETFs) on itself and they are the most liquid ETFs in the market because the investors consider those ETFs as alternative assets instead of trading the KOSPI 200 index directly. Its components are also important investment vehicles because they are qualified for the size, transparency, profitability, and business governance in the sectors.

The qualification for being a component of the index is conducted by the Korea Exchange (KRX). Based on market capitalization, sector representativeness, and other factors of the companies, its components and their weights have been regularly changed and rebalanced, respectively. For examples, when a company changes its business sector or loses large portion of sector dominance, the exchange decides whether the member in the KOSPI 200 is replaced with one of possible candidates or the composition weight of the index is modified. In addition to the regular annual changes, the current constituents can be expelled as soon as they go bankrupt or other severe unlawful activities such as dereliction of duty or misappropriation are committed.

The time period considered in our study is the twelve years span from January 2000 to December 2011. In this period, the market state has been changed including usual bull and bear markets. It also contains severe crises caused by domestic and international origins. During the period, two types of data are collected from the KRX. The first type of the data is the change log of the KOSPI 200 components including the current constituents and historical members over the period. The similar lists for its subuniverses such as the KOSPI 100 and KOSPI 50 are also obtained. All component changes are tracked and stored into a database. These change logs are really important because the survivor bias is excluded by having the complete list on the component change. Another dataset consists of historical daily data for each company. In addition to daily price information, daily fractional change, volume, total transaction value in cash, and market capitalization of each equity are downloaded. The total numbers of outstanding shares for all equities in the KOSPI 200 are easily calculated from dividing daily market capitalization by daily price.

### *3.2. Momentum/Contrarian strategy*

The validity of the physical momentum definition can be tested by comparing the returns of the physical momentum strategies with those of other physical momentum strategies over the same period. Instead of the traditional momentum strategy that uses the raw return during the lookback period as a ranking

criterion, we can construct the momentum portfolio ranked by the various definitions of the physical momentum. After finding the performance, each of the momentum strategies from the various momentum criteria are compared with others in order to measure the validity of a given momentum definition. Details on the momentum strategy will follow.

The most important variables of the momentum strategy are the length of the lookback (or estimation) period  $J$ , length of the holding period  $K$ , and sorting criterion  $\psi$ . The traditional momentum strategy uses the cumulative return during the lookback period as a ranking criterion, i.e. a triplet of the traditional momentum strategy is  $(J, K, \psi = p^{(0)})$  [12]. On the reference day ( $t = 0$ ), the cumulative returns of all instruments in the market universe during the periods from  $t = -J$  to  $t = -1$  are calculated. After sorting the instruments in the order of the ascending criterion, numbers of ranking groups are constructed and each of the ranking groups has the same number of the instruments. As an instance, if there are 200 equities and we consider 10 groups, each of sorted ranking groups has 20 equities as group constituents. Following the convention of Jegadeesh and Titman [12], the loser group who has the worst performers in the market is named as R1 and the winner group with the best performers is the last one, R10. And then the momentum portfolio is constructed by buying the winners and short-selling the losers with the same size of positions in cash in order to make the composite portfolio dollar-neutral. For the winner and loser portfolios, each group member is equally weighted in the group in which it is. The constructed momentum portfolio is held until the end of the holding period ( $t = K$ ). On the last day of the holding period ( $t = K$ ), the momentum portfolio is liquidated by selling the winner group off and buying the loser group back.

On the first day of each unit period, the momentum portfolio is constructed as explained in the previous paragraph. For examples, a weekly momentum portfolio is selected in every Monday unless it is not a holiday. Monthly portfolios are formulated on the first business day in every month. For multiple-period holding strategies, there exists overlapping period between two different strategies. The reasons of this construction are followings. First of all, the momentum return from this construction is not dependent with the starting point of the strategy formation. For example, when we implement the 12-month lookback and 12-month holding momentum strategy, construction of the portfolio occurs at the beginning of each year. Since the return results are always interfered by the seasonal effects such as January effect or others related to business cycle and taxation, it is difficult to discern the momentum effect from the seasonal effects. Second, the portfolios from overlapped periods can generate the larger numbers of return samples to fortify the statistical significance. Since the dataset here only has twelve years of historical data comparing with other studies which uses much longer time periods as datasets, its statistical significance could be lowered by the smaller size of our samples if we use the non-overlapped portfolios. Third, Jegadeesh and Titman [12] reported that there were not big differences between the returns by the overlapped and non-overlapped portfolios. Finally, the portfolio construction here can be considered as diversification which helps

to mitigate large return fluctuation of the momentum portfolio. For example, in the case of 12-month holding strategies, we have twelve different portfolios at a given moment and it is definitely diversification of the portfolio. Based on these reasons, it is more sensible that the overlapping portfolios are used in our case.

When we buy the winner and loser portfolios which provide expected returns for those groups of  $r_W$  and  $r_L$  respectively, the return by the momentum portfolio  $r_\Pi$  is simply  $r_\Pi = r_W - r_L$  because we short-sell the losers in the portfolio. When we implement the trading strategy in the real financial markets, a transaction cost including brokerage commission and tax is always important because they actually erode the trading profits. The implemented momentum return or transaction-cost-adjusted return  $r_I$  is

$$\begin{aligned} r_I &= r_\Pi - c \\ &= (r_W - r_L) - (c_W + c_L) \end{aligned}$$

where  $c_W$  and  $c_L$  are the transaction costs for winner and loser group, respectively. In general,  $c_L$  is greater than  $c_W$  because the short-selling is usually much more difficult than buying. Since the transaction cost is an one-time charge, its effect on the implemented return per unit period becomes smaller as the holding period is lengthened. Similar to other financial markets, the transaction cost in the South Korean stock market also consists of brokerage commission and tax on trade. Meanwhile, there is no tax on capital gain in the South Korean market. For an one-way trading, usual brokerage fee for online trading is from 1.8 to 2.5 bps of total sales money and offline commission is about 50 bps.<sup>5</sup> The tax is charged of 30 bps of the total sales value when the equity is sold. For a round-trip trading, 35 bps of the transaction cost look reasonable for the simulation and is considered the conservative number if the online brokerage firms are chosen as the prime brokers. Since the momentum portfolio consists of two baskets, buying and short-selling, we need to subtract 70 bps from the returns of the momentum portfolio to get our implemented returns.

When the expected return of the momentum portfolio for a given  $(J, K, \psi)$  strategy is negative, the strategy can become profitable by simply switching to the contrarian strategy  $(J, K, \psi^\dagger)$  that buys the past loser group and short-sells the past winner group, exactly the opposite position to the momentum portfolio. Contrasting to the momentum strategy following the price trend, the contrarian strategy is based on the belief that there is the reversion of price dynamics. If equities have performed well during the past few periods, investors try to sell those stocks to put the profits into their pockets. The investors who bought those equities long time ago are able to make large enough profits even when the price recently has gone slightly downward. However, buyers who recently purchased the equities might not have enough margins yet from their inventories and want not to lose money from the current downward movement

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<sup>5</sup>Each brokerage and security firm has its own commission policy. There numbers are usually for individual investors.

because of risk aversion. The only option those investors can take is just selling their holdings off. This herding behavior makes the reversion of price and it is probable to make profits from short-selling if a smarter investor knows when it would be. For the opposite case, it is also possible to buy the past losers to get advantages of using the herding because the losers are temporarily undershot by investors' massive selling force and the equities tend to recover their intrinsic values. On the way of price recovery, the short-sellers need to buy back what they sold in the past in order to protect their accounts and the serial buy-back can boost the price dynamics to the upward direction which also causes feedback that causes consequential massive buy-backs by other short-sellers. How the initial anomaly can be amplified and be grown is modeled in Shleifer and Vishny [9].

The momentum and contrarian strategies look contradictory to each other but they have only the different time horizons in which each of strategies works well. Usually, in three to twelve months scale, the equity follows the trends [12] but the reversal effect is dominant at the longer and shorter scales than the monthly scale [20, 47]. For the contrarian strategy, the portfolio return  $r_{\Pi^\dagger}$  is given by

$$r_{\Pi^\dagger} = r_L - r_W = -r_\Pi.$$

The transaction cost adjusted return  $r_I$  for the contrarian strategy is

$$\begin{aligned} r_I &= r_{\Pi^\dagger} - c \\ &= (r_L - r_W) - (c_W + c_L). \end{aligned}$$

When implementability of a given strategy in the real markets is the main concern, we need to focus on whether or not it is possible to take actual profits from the strategy among the momentum and contrarian strategies. In this sense, the profitability of the strategy with absolute (implemented) return  $\tilde{r}_I$  can be measured by

$$\tilde{r}_I = |r_W - r_L| - (c_W + c_L)$$

and tells whether the potential trading profit can exceed the barrier of the transaction cost. The actual positive return from the momentum/contrarian trading strategies can be taken into the pocket when  $\tilde{r}_I$  is positive.

As mentioned above, the method for measuring the price momentum is the momentum strategy with the physical momentum as a ranking criterion. There are total eleven types of candidates for physical momentum including the original cumulative return momentum. On the reference day ( $t = 0$ ), each physical momentum for equities over the estimation periods is calculated and is used for sorting the equities. The ranking for each criterion constructs the momentum portfolios. After holding the portfolio during the given period, it is liquidated to get the momentum profit. The positive implemented returns and Sharpe ratios from implemented return exhibit the robustness of the physical momentum strategies. If their returns beat that of the traditional momentum strategy, it is

obvious that the physical momentum definition really has a merit to introduce and there is a practical reason to use the momentum strategies based on the physical momentum as an arbitrage strategy.

For the lookback period, some stocks which don't have enough trading dates are ignored from the analysis. In general, this case happens to companies which are enlisted to the KOSPI 200 amid of the lookback period. If an equity is traded on only one day during the estimation period, it is neglected from our consideration for the momentum strategy universes because it is impossible to calculate the standard deviation for  $p^{(3)}$ -type momentum for these stocks. Since all possible candidates for the physical momentum need to be compared with other criteria over the same sample, it is obvious not to consider these equities with only one trading day in the estimation period. The companies delisted amid of the holding periods don't cause the same problem because only the lookback return is important in sorting the equities and constructing the momentum portfolios. In this case, the returns for the delisted companies are calculated from the prices on the first and last trading days in the holding periods.

## 4. Results

For brevity, we represent only four results, one from each category of the physical momentum definitions including the original cumulative return momentum,  $p^{(0)}$ . This omission of part of the results is guaranteed by the fact that the profitability, return, volatility, and Sharpe ratio heat map patterns of a given momentum definition are similar to the results of the other definitions in the same category over the whole strategy spaces,  $12 \times 12$  lookback-holding pairs, although minor differences and exceptions exist. This similarity in the patterns seems to be based on how to define the physical momentum. The choices between the normal return and log return or between the fractional volume and transaction amount in cash don't bring any big differences in the patterns but the categories of the physical momentum definition make rather clearer cuts in characteristics of the results. Among various definitions, the momentum that uses the fractional volume as mass and log return as velocity is chosen for our graphical representation and analysis because the log return is exactly the precise definition of the financial velocity than the raw return.

### 4.1. KOSPI 200

#### 4.1.1. Weekly strategies

First of all, let's review the best strategies of our constructed portfolios. Without consideration on the transaction cost, the best strategy from the raw return momentum  $p^{(0)}$  in the KOSPI 200 market pool provides the return of weekly -1.39% at 1/1 of which the minus sign tells that the contrarian strategy works well as expect in Lo and McKinlay [20]. The t-value for the best strategy is -10.29 which means 1% level of significance (99% confidence level) under a null hypothesis that the expected return is zero and the null hypothesis is rejected. The weekly Sharpe ratio is -0.412. These numbers are similar to

the weekly momentum results from the previous studies by Choi [35] with the dataset of 2000-2010 and by Choi et al. [48] with the dataset of 2000-2011 including all equities in the KOSPI 200. As mentioned above, the dataset here neglects equities which don't have enough numbers of data points during the lookback periods. However, the slightly-modified market pool with respect to the previous studies doesn't give any serious impact on the final momentum profitability. Although there are several major differences in the dataset, the fact, that momentum returns are almost identical to the results in Choi [35] and Choi et al. [48], imposes that the robust momentum effect still exists in the KOSPI 200 market. It is also similar that the larger portion of the portfolio returns comes from the loser group.

Under the same condition, the physical momentum strategies are also profitable and the Sharpe ratios exhibit that the portfolios have the stable performance although they are not as good as the traditional momentum strategy. The returns by the physical momentum strategies are all significant at least in 95% confidence level. The  $p^{(1)}$ -related criteria provide slightly weak returns which are all contrarian. The returns are in the range of 1.16%–1.18% and the volatilities from four  $p^{(1)}$ s are all around 3.50%. Their Sharpe ratio are in the range of -0.32 and -0.33. The returns of the  $p^{(2)}$  momentum strategies, 1.24%–1.25% are better than  $p^{(1)}$  in performance but also weaker than the cumulative momentum. However, their volatilities are around 3.34% which are smaller than the  $p^{(0)}$ - and  $p^{(1)}$ -based criteria. The Sharpe ratios are in the range of 0.368 and 0.371. The  $p^{(3)}$  returns, about 1.09%, are the worst ones among all candidates but the volatilities around 2.2% also have the smallest values. The Sharpe ratios of  $p^{(3)}$  momentum strategies are -0.383 and -0.377.

However, the overall parameter spaces for the strategies need to be covered because looking at the best strategy only gives part of information. For examples, a given physical momentum definition might have a dominant peak at a certain point on the  $J/K$  parameter space and shows the poorer performance elsewhere. In this case, it is not easy to decide whether the best performance is created by the momentum effect or by data errors. If we accept this peak as the best performer, the best strategy for the physical momentum exaggerates the validity and performance of that definition and it leads to the distorted conclusion on the robustness of the physical momentum. It is called data snooping and more numbers of the lookback-holding pairs need to be considered in order to prevent the data snooping.

Profitability results of the physical momentum strategies are given in Fig. 1. The profitability tells whether a certain strategy has positive or negative expected returns. The transaction cost is not considered yet. If it is positive, the momentum strategy is executed and the negative return leads to the contrarian strategy. The  $p^{(1)}$  shows the reversal over all weekly strategies. However,  $p^{(2)}$  and  $p^{(3)}$  behave much similar to the cumulative return based momentum strategy at the upper-right corner which means long-term weekly strategies. It is dominated by the reversal in the short-term region but becomes the trend-following in the longer-term region. With any definitions, we need to use the contrarian strategy in order to take a profit with small  $J$ s and  $K$ s.

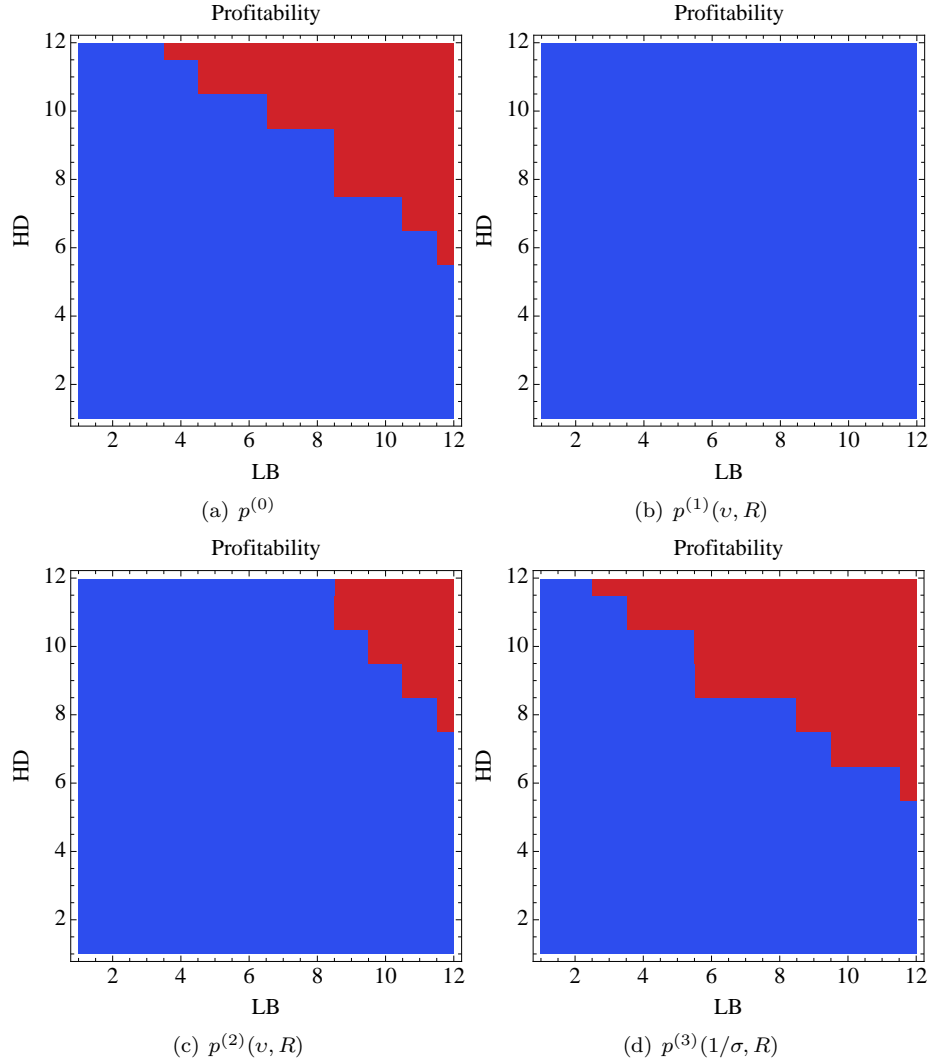


Figure 1: Profitability heat map of weekly strategies. The x-axis is for lookback and y-axis for holding period. The negative expected return at a given strategy corresponds to blue and the positive returns give red.

The heat maps for the implemented returns are given in Fig. 2. It shows the expected return when the given strategy is implemented in the real market with transaction cost of 0.35%. Even though some strategies have positive expected returns without the transaction cost, it might not be profitable because of transactional cost as market friction. The same situation also happens to the contrarian strategy. The details are given in Table 1.

The  $p^{(0)}$  has the non-implementable strategies located along a diagonal line



Table 1: Strategy and P&L based on profitability and implemented return			
Profitability	Implemented return	Strategy to use	Profit&Loss
Positive	Positive	Momentum strategy	Profit
Positive	Negative	Momentum strategy	Loss
Negative	Positive	Contrarian strategy	Profit
Negative	Negative	Contrarian strategy	Loss

around the intermediate lookback and holding periods. This is well-matched to the profitability of  $p^{(0)}$  in Fig. 1. Around the similar position, the profitability experiences smooth transition between the contrarian in the shorter terms and the momentum in the longer terms. Since the strategies in this region don't perform strongly enough to beat the transaction cost in any given directions, the implemented returns after subtracting the transaction cost are negative values. In real markets, it is much better to stop execution of the strategies not to lose the money.

However, the heat map for the implemented return of  $p^{(1)}$  has the totally different pattern with that of  $p^{(0)}$ . The strategies with  $p^{(1)}$  have positive values at almost all pairs of the lookback and holding periods, except for some short-term strategies. This pattern exhibit that when the strategies based on  $p^{(1)}$  are used as trading strategies in the real markets, positive returns are gained from the contrarian strategy and the expected returns don't have strong dependence on selection of the lookback and holding periods. This observation has the important meaning that the strategies based on  $p^{(1)}$  show stability of the performance that protects the portfolio from losses caused by sudden changes of the optimal strategy in the future. The  $p^{(2)}$ -strategies are similar to  $p^{(0)}$  but the area of positive values are different with that of  $p^{(0)}$ . Contrary to the previous two categories, the  $p^{(3)}$  momentum performs poorly because negative implemented returns at most of lookback-holding sites are obtained. In many cases, these  $p^{(1)}$ -,  $p^{(2)}$ -, and  $p^{(3)}$ -based physical momentum strategies have statistically significant returns at 5% or 1% significance.

The volatility is also evidence for the effectiveness of the physical momentum. The volatility heat map from each physical momentum definition shows that the volatility for a given definition fluctuates in the narrower range than that of the cumulative return does. Most of them are roughly in the range from weekly 2.9% to 3.5% while the volatility of the traditional momentum varies from 3.2% to 4.2%. In particular, the volatility of  $p^{(2)}$  only varies between 2.9% and 3.1%. In addition to that, the volatilities of  $p^{(1)}$  and  $p^{(2)}$  don't depend on the choice of the lookback and holding periods similar to the return results. Meanwhile, the volatilities by  $p^{(0)}$  and  $p^{(3)}$  become larger as the lookback period are extended. These observations on the volatility imposes that the physical momentum strategies by  $p^{(1)}$  and  $p^{(2)}$  actually provide the consistent and stable returns regardless of the lookback and holding periods.

In order to check the robustness of the physical momentum over the tradi-

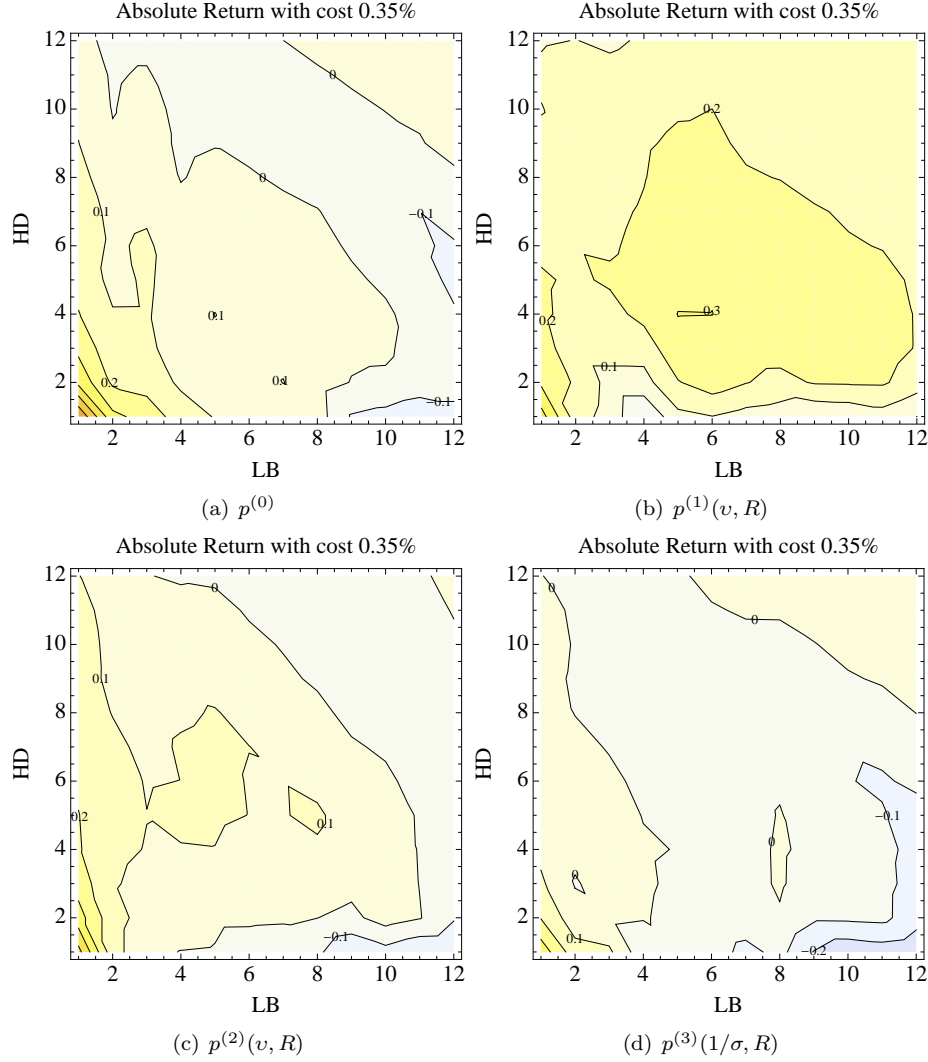


Figure 2: Weekly absolute returns heat map of the physical momentum strategies with transaction cost of 0.35%. As closer to 1%, it become more red and turns to blue as closer to -1%.

tional momentum, two statistics need to be compared. They are relative return and Sharpe ratio differences between the physical and traditional momentum strategies. The results are given in Fig. 4. In the cases of  $p^{(1)}$  and  $p^{(2)}$ , the physical momentum seems to be a good selection variable for construction of the portfolio. The both categories outperform the cumulative return based strategy in relative performance strength and Sharpe ratio at most of the lookback and holding pairs. In particular,  $p^{(1)}$  has the better performance and lower risk

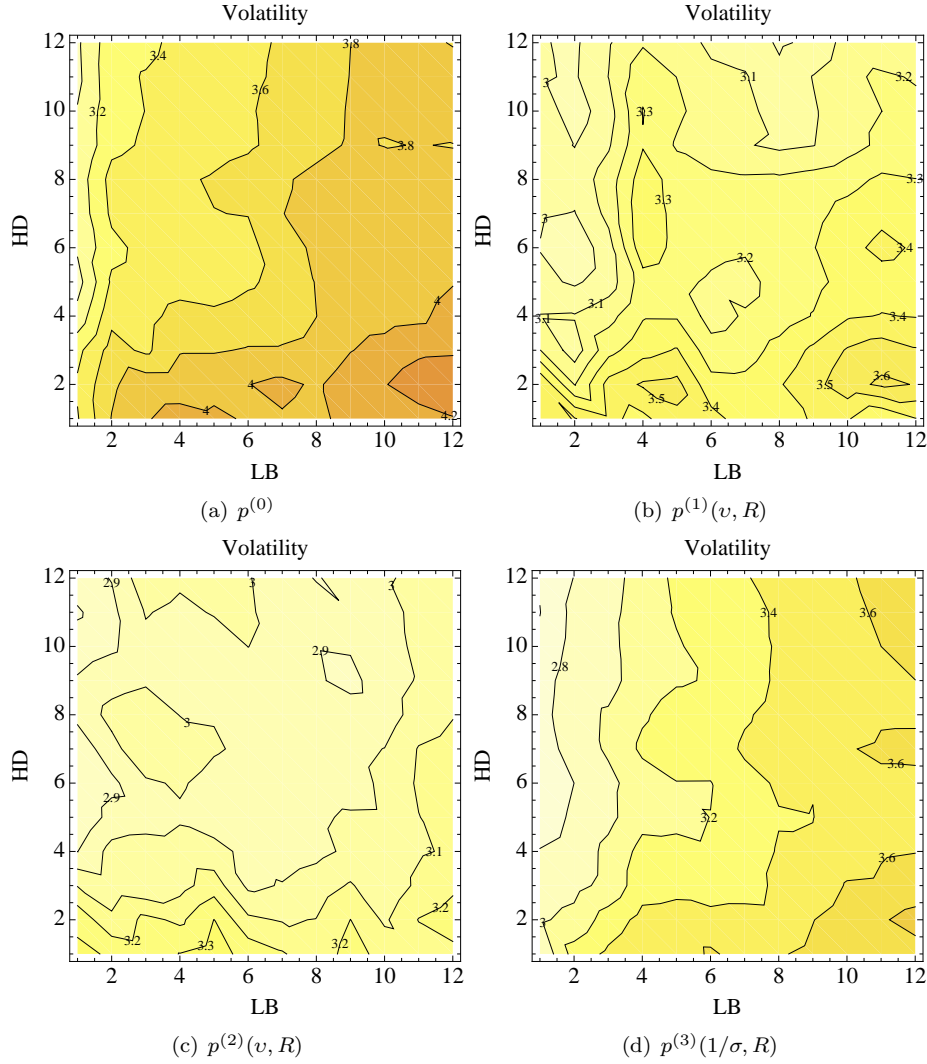


Figure 3: Weekly volatility heat map of the physical momentum strategies, As closer to 5%, it become more red and turns to blue as closer to 0%.

than any other momentum strategies. However,  $p^{(3)}$  doesn't have any merits to use because its relative return and Sharpe ratio are poorer than those of the traditional momentum strategy. The outperformance of  $p^{(1)}$  and  $p^{(2)}$  can be related to the results in Lee and Swaminathan [42] that although the lower volume stocks predict the higher future returns, the momentum strategies work better among high volume stocks. The larger volume and higher past return stocks have the larger positive physical momentum than the stocks with lower volume and higher past returns. Meanwhile, the large volume but poor past

return equities have the smaller physical momentum than the small volume and poorly performed equities. This is the relation how the volume-based momentum strategy by Lee and Swaminathan [42] has a connection with the physical momentum strategy in this paper.

The possible explanation on the fact that patterns from  $p^{(1)}$  are similar to those from  $p^{(2)}$  and they have different patterns with  $p^{(0)}$  and  $p^{(3)}$  can be found in the way how to define the physical momentum. The  $p^{(1)}$ - and  $p^{(2)}$ -momentum are from the summation of daily momentum fluctuation although minor differences in mass and velocity exist. They tend to record the daily fluctuation which can detect more information on the equity price. Meanwhile,  $p^{(0)}$  and  $p^{(3)}$  contain the cumulative return in their definition. If we multiply the number of days in the lookback period, the Sharpe ratio is changed to the raw return divided by the volatility. Since most of equities have the same number of trading days, the only main difference between each equity is the volatility. In addition to that, the normalization in the  $p^{(2)}$  definition also gives weak impact on the final result. Although both of  $p^{(1)}$  and  $p^{(2)}$  outperform,  $p^{(1)}$  has the stronger performance over all lookback-holding sites but the patterns of the  $p^{(2)}$ -strategy are much similar to the  $p^{(0)}$  and  $p^{(3)}$ -strategies.

#### 4.1.2. Monthly strategies

The  $p^{(0)}$ -momentum strategy provides monthly 2.48% with the 9/6 strategy as the best strategy among 144 strategies. The t-value for the best performance is 2.32 corresponding to 95% confidence level with the same null hypothesis of the weekly case. The monthly standard deviation of the best strategy is 12.18% and the monthly Sharpe ratio is 0.204. Similar to the weekly strategies, these numbers are not largely different with the results in Choi et al. [48] which uses all equities whether or not they have enough numbers of trading dates in the lookback periods. However, the momentum returns herein and results given in Choi et al. [48] are different with other studies [28, 29, 30, 32] that reports no sign of existence of the momentum effect in the South Korean markets. The main reason of mismatch is that those studies used all the equities in the KOSPI but the first two studies used the KOSPI 200 which is part of the KOSPI.

The best  $p^{(1)}$ -based momentum strategies are governed by the reversion of price. The four  $p^{(1)}$  criteria have negative momentum returns between -1.75% and -1.5%, i.e. the contrarian strategy works well. Additionally, their Sharpe ratios are between -0.259 and -0.228 and the absolute values of the Sharpe ratios are larger than that of the cumulative return based momentum strategy. In particular,  $p^{(1)}(v, r)$  provides monthly -1.75% with the 2/1 strategy of which the t-value is -3.09, 99% confidence level. Its Sharpe ratio is -0.259, about 25% times larger in absolute magnitude than that of the traditional momentum strategy. This increased Sharpe ratio is caused by the much smaller volatility of 6.73% comparable with 12.18% by the cumulative return criterion. The momentum portfolio based on the physical momentum has a merit to construct because each of winner and loser groups has the larger volatility than that of the composite portfolio. The other  $p^{(1)}$  momentum strategies also have the similar results.



Opposite to the  $p^{(1)}$  cases, the best  $p^{(2)}$  momentum strategies show the trend-following. They have the best strategies at 7/7, 11/3, 11/3, and 9/5 which are relatively longer than the 2/1 of the  $p^{(1)}$ -strategies. The returns in the range between 1.13% and 1.45% become smaller than the  $p^{(1)}$ -based momentum strategies but they still have the comparable Sharpe ratios with the cumulative return based momentum except for  $p^{(2)}(\tau, r)$ . All the strategies from  $p^{(2)}$  have the smaller momentum volatilities, 6.36%–6.89% than those of the winner or loser portfolios. The best  $p^{(3)}$  momentum strategies have different patterns than others. Their returns are comparable with that of the cumulative return based momentum.  $p^{(3)}(1/\sigma, r)$  at 7/7 has the larger momentum volatility which imposes the smaller Sharpe ratio. Its momentum volatility is greater than the volatilities from the winner and loser groups. However,  $p^{(3)}(1/\sigma, R)$  at 11/4 has the better Sharpe ratio than the raw criterion and the volatility of the strategy is smaller than those of the winner or loser portfolios.

Similar to the weekly strategies, the overall lookback-holding pairs need to be considered. The monthly strategies have different characteristics with the weekly strategies. In Fig. 5, the profitability of the raw momentum strategy,  $p^{(0)}$ , shows the reversal in the short terms and trend-following in the long terms. The similar aspects are observed from the  $p^{(2)}$ - and  $p^{(3)}$ -based momentum strategies although the area of negative values varies. Its profitability is also not dependent with the definition in the category. However,  $p^{(1)}(v, R)$  shows the reversal at most of pairs and its trend-following strategies don't have any regular border. In addition to that, the profitability pattern of  $p^{(1)}$  varies with respect to the definition. When the normal return is used for the velocity, the profitability from each mass definition looks very similar to another. However, the log return doesn't have any similarity.

The similar results are found in Fig. 6 for the heat map of the implemented returns. In the monthly scales,  $p^{(0)}$ ,  $p^{(1)}$ , and  $p^{(3)}$  are similar to each others. Their patterns have the peaks along the diagonal line near the intermediate and long term region. In the short terms, the returns from them become negative and it means that the returns at those lookback-holding periods are not strong enough to beat the transaction cost with comparing the profitability result in Fig. 5. However, the  $p^{(1)}$  returns behave differently with other momentum categories. There is no dominant peak in the return heat map. It shows the relatively constant returns at all the pairs of the lookback and holding periods. The negative valued region is well-matched to the heat map for profitability in Fig. 5.

The volatilities of the physical momentum strategies in Fig. 7 are divided into two groups. The first one includes the  $p^{(0)}$ - and  $p^{(3)}$  momentum strategies and its pattern has the peak. Meanwhile, the volatilities of  $p^{(1)}$  and  $p^{(2)}$  are relatively constant over the whole parameter spaces comparing with the  $p^{(0)}$  and  $p^{(3)}$  cases. These patterns are identical to the volatility patterns of the weekly momentum strategies. Similar to the weekly strategies,  $p^{(1)}$  and  $p^{(2)}$  provide the smaller return volatilities which are helpful to construct the more stable portfolios.

Contrary to weekly strategies, the monthly physical momentum strategies

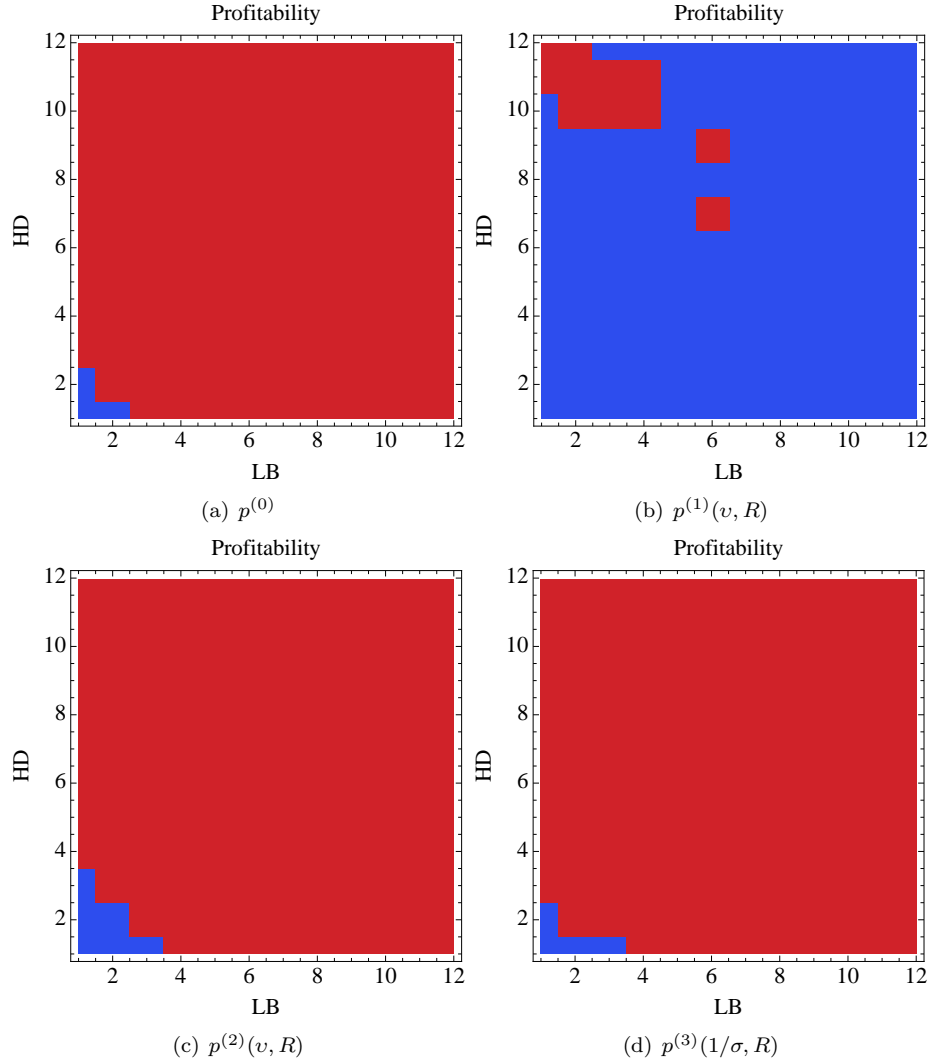


Figure 5: Profitability heat map of monthly strategies. The x-axis is for lookback and y-axis for holding period. The negative expected return at a given strategy corresponds to blue and the positive returns give red.

are not as good as the traditional momentum strategies. Their relative returns and Sharpe ratios are given in Fig. 8. All of the physical momentum strategies show the weaker performances and smaller Sharpe ratios than the raw return momentum. However, in the short terms up to five or six months,  $p^{(1)}$  and  $p^{(2)}$  exceed the  $p^{(0)}$ -momentum strategy. This bound can cover the maximum time horizon of the weekly strategy, twelve weeks and it is observed that  $p^{(1)}$  and  $p^{(2)}$  have the stronger performances in those weekly scales. From these facts,

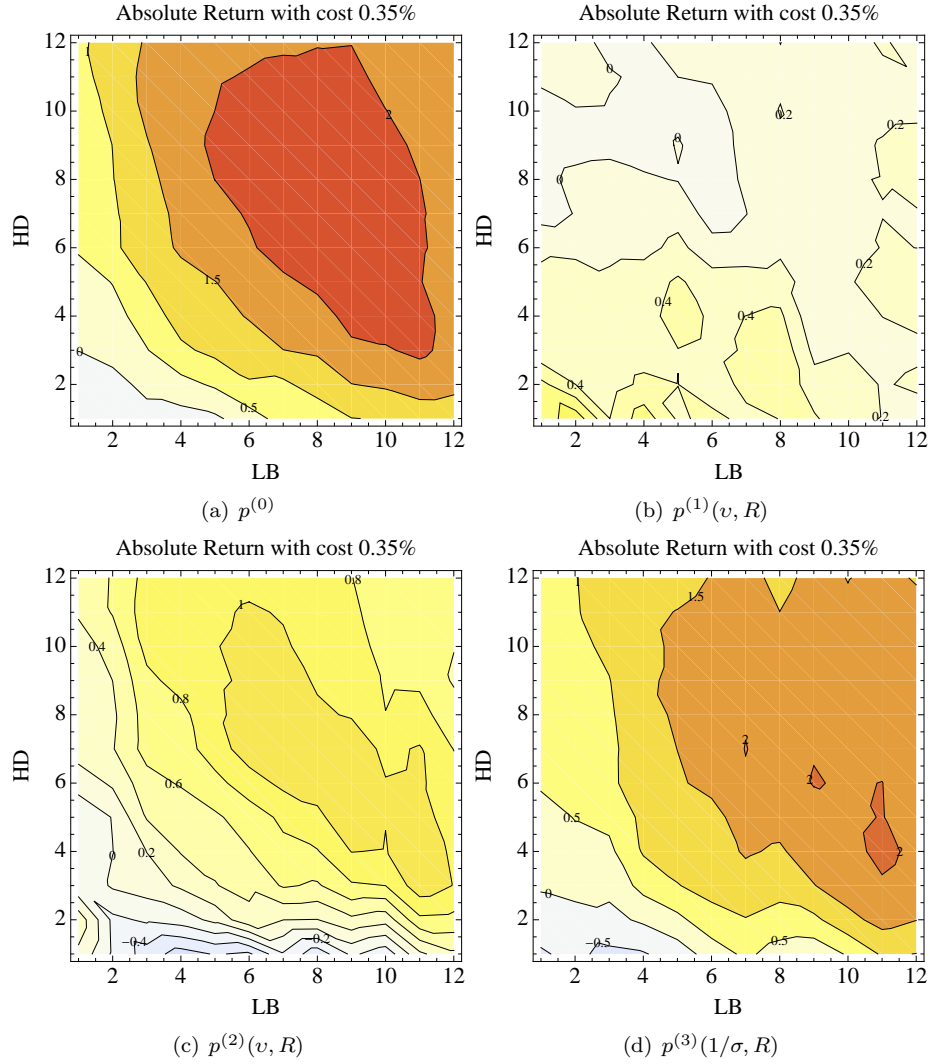


Figure 6: Monthly absolute return heat map with transaction cost of 0.35% for the momentum strategies. As closer to 2.5%, it become more red and turns to blue as closer to -2.5%.

it is guessed that the physical momentum has the effective range of the time horizon. Below the time bound, the physical momentum can incorporate any information which can be helpful to forecast the future returns. However, the more noise signals contaminates any meaningful information which the physical momentum definitions encode as the time horizon is extended.



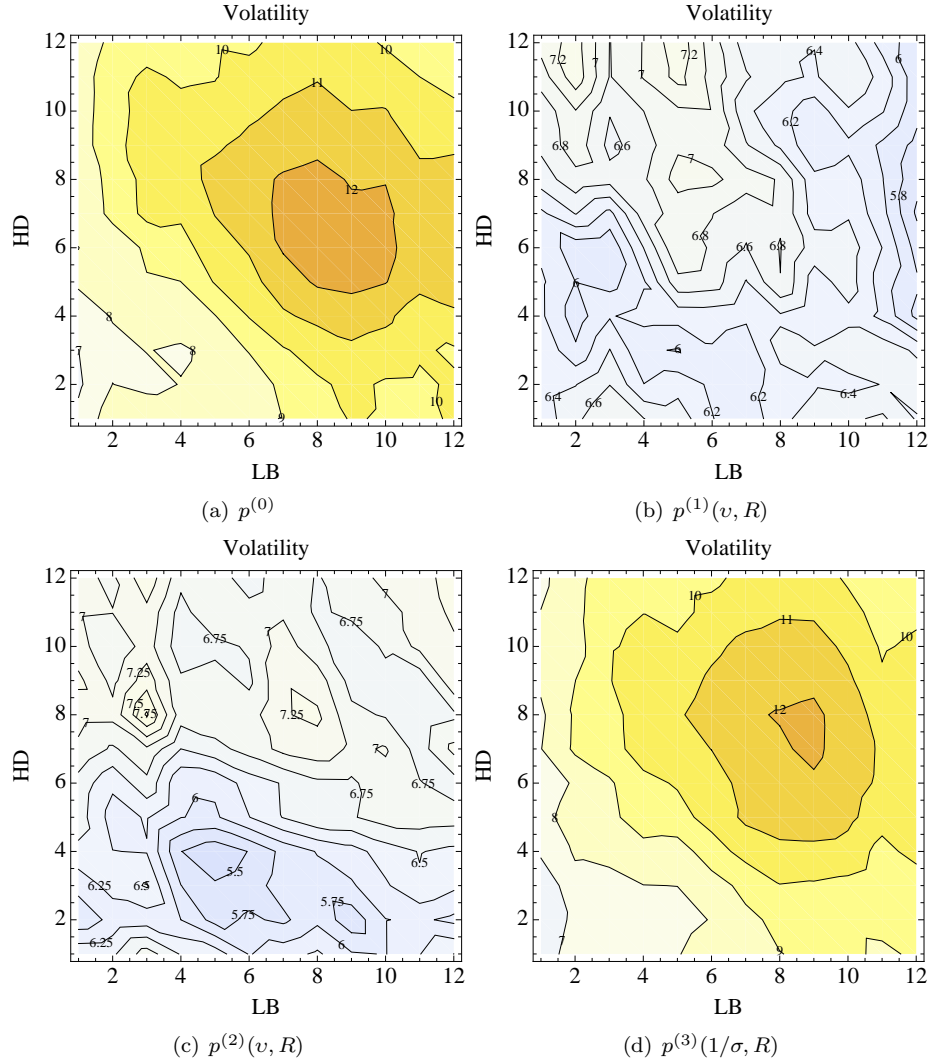
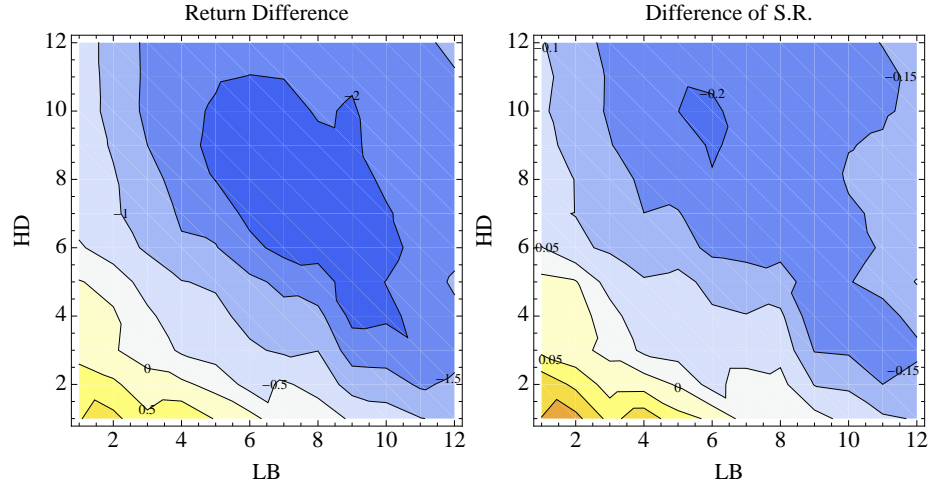


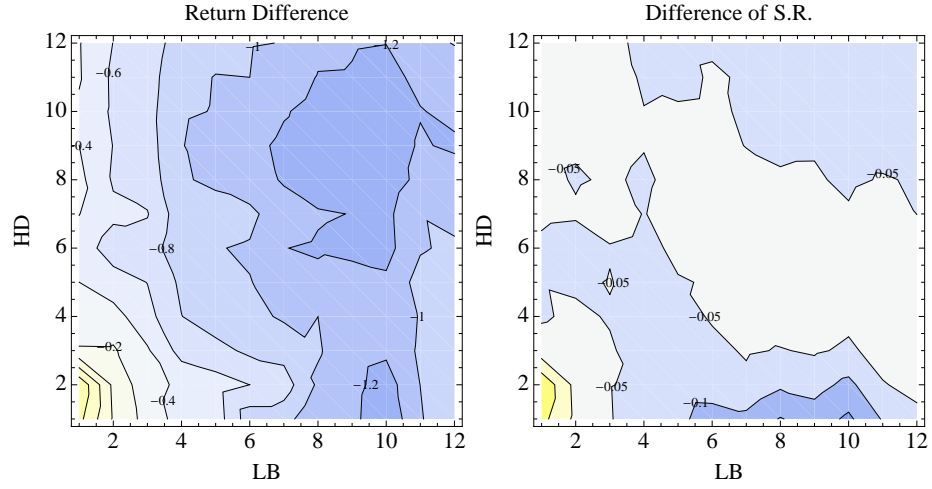
Figure 7: Weekly volatility heat map of the physical momentum strategies, As closer to 15%, it become more red and turns to blue as closer to 0%.

#### 4.2. Other sub-universes

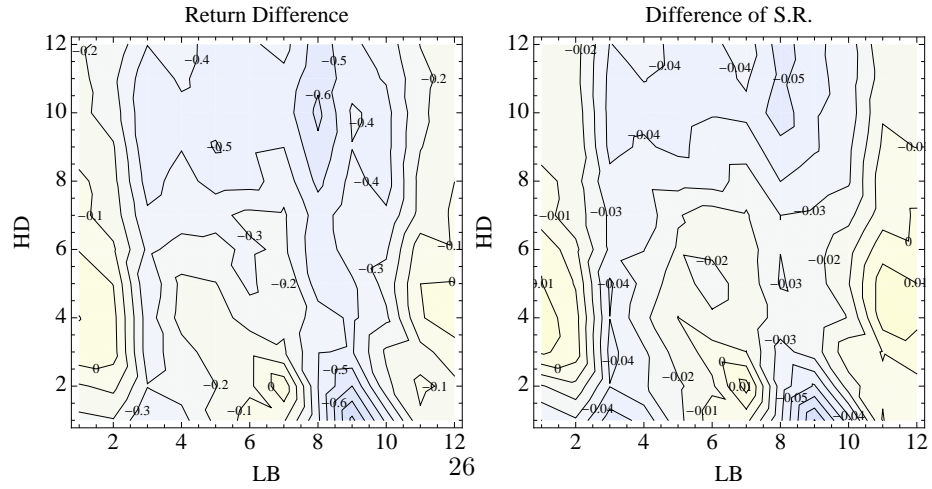
When the market universe for the momentum strategy is changed, it is observed that the momentum return varies with respect to the universe. Besides the magnitude of the momentum return, the profitability is also changed [48]. In order to check the consistency of the validity on the physical momentum definitions, it is necessary to repeat the implementation of the momentum strategies in other subuniverses of the KOSPI 200. With the convention in the work on market universe dependence [48], we simulate the momentum strategies in order



(a)  $p^{(1)}(v, R)$



(b)  $p^{(2)}(v, R)$



(c)  $p^{(3)}(1/\sigma, R)$

Figure 8: Difference of Return and Sharpe ratio between physical momentum strategies and the original momentum strategy in monthly scale. The red corresponds to 2.5% for return and 0.25% for Sharpe ratio.

to test the robustness of the physical momentum strategies.

One important caveat is that the physical momentum is also valid in other subuniverses of the KOSPI 200. For example, the patterns of return, volatility, and Sharpe ratio by the physical momentum in different universes are similar to the results by the KOSPI 200 case. For the weekly strategies, the physical momentum strategies in a certain market pool become more profitable than the traditional momentum strategy in the market. The Sharpe ratios are also better than that by the cumulative return momentum strategies. When the time scale for the strategy is lengthened to the monthly scales, the physical momentum strategies only beat the traditional momentum strategy in the short terms less than three months.

While the original definition of the price momentum provides different profitabilities and returns as the market universe is changed, the physical momentum sustains the structure of the momentum return and remains consistent and meaningful over the change of market universes. The fact that the patterns observed in the KOSPI 200 remain the same with small variations and exceptions regardless of the market universe change means the validity and effectiveness of the physical momentum introduced in the paper. This also impose the robustness of the physical momentum.

## 5. Test for symmetry breaking of arbitrage

In the frame of spontaneous symmetry breaking (SSB) of arbitrage suggested by Choi [35], profitable trading strategies are considered as being in the symmetry breaking mode which is triggered by a control parameter. Although the brief introduction to the SSB of arbitrage is given here, see the original paper [35] and references therein for more details. The arbitrage dynamics is modeled by a stochastic differential equation

$$\frac{dr(t)}{dt} = -(\lambda - \lambda_c)r(t) - \lambda_c \frac{r^3(t)}{r_c^2} + \nu(t)$$

where  $r(t)$  is the arbitrage return of trading strategies and  $\lambda$  is the control parameter.  $\lambda_c$  and  $r_c$  are considered as constants and in particular,  $\lambda_c$  is a cut-off for phase transition.  $\nu(t)$  is a stochastic term which generates a random walk and a probability distribution of the random walk is not specified yet.

For a stationary state in the long run, there are several solutions for the stochastic differential equation. In the case of  $\lambda \geq \lambda_c$ , a solution is  $\langle r \rangle = 0$  which corresponds to a no-arbitrage state. This phase is expected by the no-arbitrage theorem because the arbitrage trading of which the expected return is non-zero cannot be possible to exist. Meanwhile, there are also non-zero solutions if  $\lambda < \lambda_c$ . These solutions are exotic to the no-arbitrage theorem. The SSB returns are given by

$$\langle r \rangle = \pm \sqrt{1 - \frac{\lambda}{\lambda_c}} r_c = \pm r_v$$

and the sign is not important here because the portfolio position can be inverted by short-selling in order to take the positive profits. Additionally, since all trading strategies are devised for making the positive returns, the external field in the SSB, if exists, which prefers one of the non-zero values, can be thought to lead the solution to choose the positive return.

With the SSB, the portfolio can be executed under the following scheme. First of all,  $\lambda$  and  $\lambda_c$  for the next step are estimated from the time series of the past arbitrage returns and the benchmark returns, respectively. To estimate the future  $\lambda$ s, an autocorrelation coefficient is used and the estimated parameter  $\hat{\lambda}$  is given by

$$\hat{\lambda}_{i+1,k} = 1 - \frac{\langle r_i r_{i-1} \rangle_k}{\langle r_i^2 \rangle_k}$$

where the first term comes from the discretization of the stochastic differential equation. It is guaranteed by the fact that the arbitrage dynamics could be approximated to an autoregression model (AR) of order 1. The parameter of the AR(1) is the autocorrelation coefficient because  $|r| \ll 1$  makes the third term in the arbitrage dynamics ignorable. After estimating the  $\lambda$ s for the next periods, the both parameters need to be compared in order to decide whether or not the strategy is in the phase of arbitrage. When  $\hat{\lambda} < \lambda_c$ , the strategy will be executed because it is expected to be in the arbitrage mode. Meanwhile, the strategy will be stopped elsewhere.

The algorithm is applied to the physical momentum strategies in the same set of the market universes we used in the previous section and the conclusions are followings. First of all, similar to the observations for the physical momentum/contrarian strategies in the previous section, the results from the same category give the similar patterns on the average returns and Sharpe ratio of the SSB-guided strategies without exceptions. Secondly, in almost all strategies, the SSB of arbitrage provide the better performance than the original strategies without the SSB algorithm. In addition to that, the patterns of the returns and Sharpe ratios are close to the patterns observed in the previous study [35] that the algorithm performs well in short MA horizons, the magnitude of performance becomes smaller in the intermediate time scales, and it slightly recovers the effectiveness or stagnates in the long run. The only exceptions are the physical momentum strategies in KOSPI (100-50). In that universe, there are no short-term hikes in returns and Sharpe ratios by all momentum definitions but they are restored in the long-term horizon.

Based on the previous findings, the results over only the KOSPI 200 components are given for brevity. Similar to the physical momentum strategy case, one result from each category are represented in the paper. Additionally, we test the 1/1 weekly strategies to compare with the previous result in Choi [35] which used the 1/1 weekly strategy. The returns of the SSB-guided physical momentum strategies are given in Fig. 9. All of the returns are improved by the SSB scheme. They are much better in the short MA windows for  $\lambda$  calculation. It is exactly identical to the previous observation in Choi [35]. In particular, the SSB scheme works well in the case of  $p^{(1)}(v, R)$ .

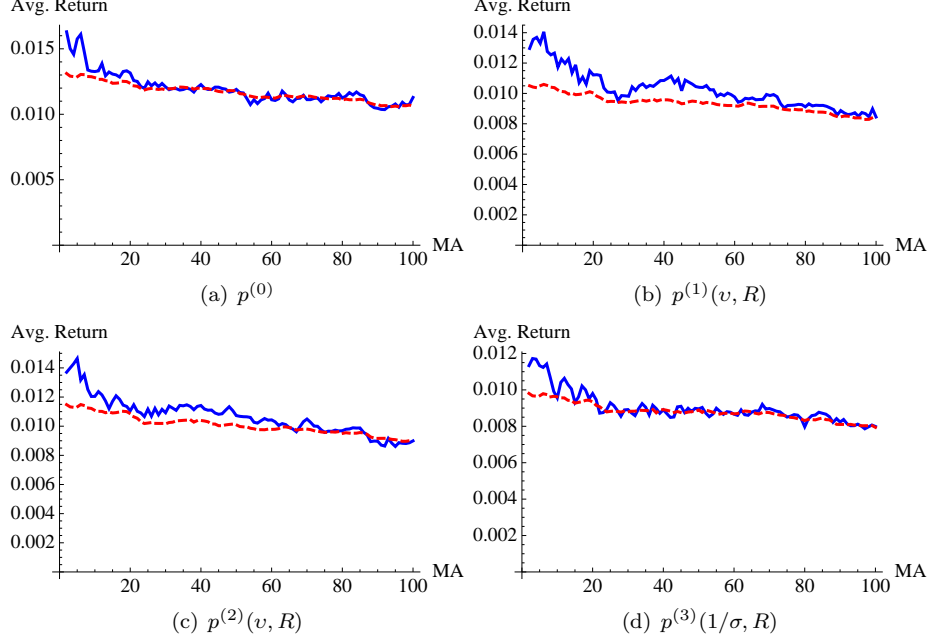


Figure 9: Returns of SSB-aided physical momentum strategies (blue) and the physical momentum strategies without SSB (red dashed). The MA window size ranges from 2 to 100. The y-axis is in return not in percentage.

In addition to the return, the Sharpe ratio given in Fig. 10 has the similar pattern to the cumulative return result. The Sharpe ratios for the physical momentum strategies under the symmetry breaking idea are larger than those of the physical momentum without the SSB. The Sharpe ratios become much greater in the short-term region and tend to be better over all MA windows. This pattern is also observed in the traditional momentum strategy [35].

The similar results are obtained from other universes such as the KP100, KP50, and other complementary subsets. Most of them have the better performance in the return and Sharpe ratio when the SSB scheme is used. In particular, the short estimation period for  $\lambda$  calculation brings much better results than the long-sized windows. These patterns are also identical to the result in the SSB. In this sense, the SSB of arbitrage is capable of guiding the physical momentum strategies to more lucrative and more stable strategies. Small numbers of slightly different patterns in the returns or Sharpe ratios are also found but only some physical momentum categories are in those cases. For these exceptions,  $p^{(1)}$  and  $p^{(3)}$  in the KOSPI (100-50) don't have strong returns and Sharpe ratios in small sized MA windows.

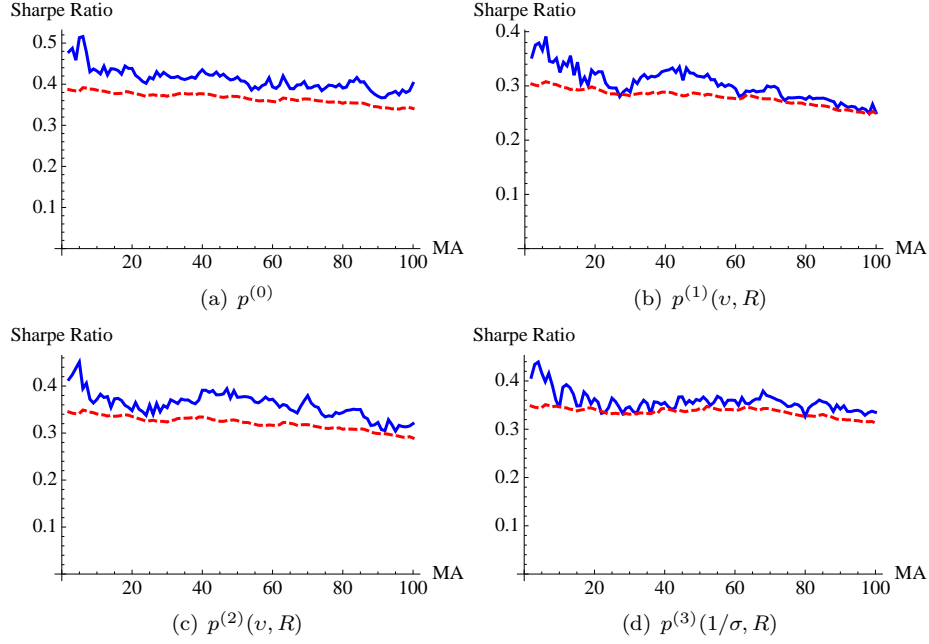


Figure 10: Sharpe ratios of SSB-aided physical momentum strategies (blue) and the physical momentum strategies without SSB (red dashed). The MA window size ranges from 2 to 100.

## 6. Conclusion

In this paper, the various definitions of the physical momentum on equity price are introduced. Using the mapping between the price of a financial instrument and position of a particle in the one dimensional space, the log return corresponds to the velocity in equity price space. Up to the higher-order correction terms, the cumulative return is also considered as the velocity. The candidates for the financial mass to define the equity momentum quantitatively are the fractional volume, fractional transaction amount in cash, and the inverse of volatility. These definitions have the plausible origins not only from the viewpoint of physics but also from finance.

With the financial mass and velocity concepts, it is capable of defining the physical momentum in price that is called as the price momentum in finance. Measuring the physical momentum for each equity in the KOSPI 200, the main index of the South Korean market, the momentum strategy which uses the physical momentum as a ranking criteria is implemented. Its performance and risk-reward ratio surpass those of the traditional momentum strategy in the weekly level. For the shorter terms in monthly scale, the physical momentum strategies also exceed the raw momentum strategy. Since the shorter months are in the range of the weekly levels up to twelve weeks, these observations imposes that there exists the proper length of the time horizon which can incorporate the information on forecast of future price change with the physical momentum.

The more interesting observation is that the physical momentum strategies also outperform the traditional momentum strategy in other market universes which are the subsets of the KOSPI 200. Testing over six different subsets of the KOSPI 200, the similar patterns with those for the KOSPI 200 are obtained in the weekly levels. While the performance of the traditional momentum strategy changes severely as the market universe is altered, the performance and Sharpe ratio patterns by the physical momentum strategies beat the traditional momentum strategy although a few exceptions exist. It imposes the ubiquitous existence of the physical momentum.

In addition to the performance, the idea of symmetry breaking arbitrage also works well for the physical momentum strategies. Estimating the control parameter  $\lambda$  and critical value  $\lambda_c$  for phase transition, the scheme, that executes the strategy if  $\lambda < \lambda_c$  and stops the execution elsewhere, improves the performance and stability of the physical momentum strategies. Moreover, the patterns of the improved returns and Sharpe ratios are identical to the previous study [35]. The invariant patterns are also found in cases of the physical momentum strategies in other market universes which are the subsets of the KOSPI 200.

In future work, the same test will be conducted in other markets such as the U.S. stock markets and in different trading strategies including high frequency tradings. Additionally, factor analysis with the physical momentum will be considered to explain the origins of the physical and traditional momentum. Many literatures in finance have tried to explain the momentum profits in the framework of factor model. Although the Fama-French three factor model failed to find the origin of the returns, introduction of the momentum factor to mutual fund performance explains part of unanswered questions [49]. In the similar way, it is possible to understand the momentum profits with consideration on the physical momentum factor.

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